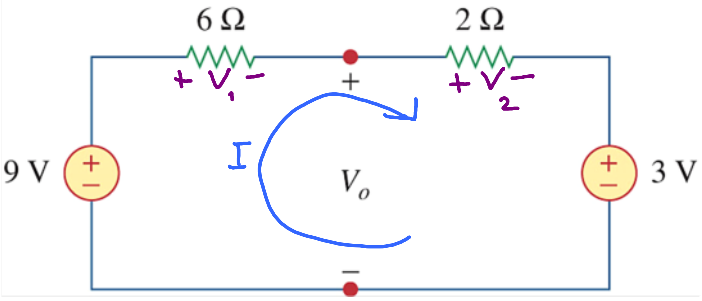
[Alexander and Sadiku, 2009, Q2.16]

Monday, June 10, 2013 7:59 PM

First, note that this circuit is similar to the circuit in Example 2.3.4 which we discussed in class. All elements are connected in series forming a single loop.



KCL says that we can use one current I throughout the whole loop.

Next, we define the voltages V_1 and V_2 as shown in the figure. Note that their polarities can be arbitrarily selected. However, we define then such that they conform with the passive sign convention when the direction of I is also considered.

Now, by KVL (over this single loop), $+9 - V_1 - V_2 - 3 = 0$

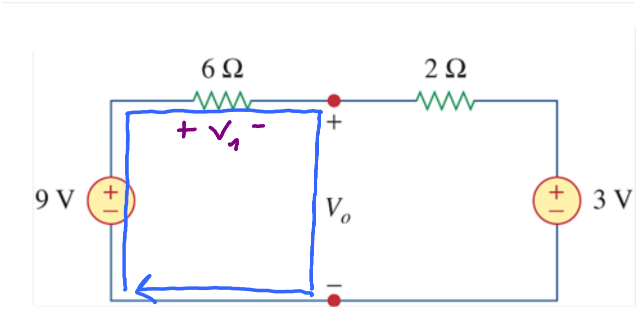
Also, by Ohm's law, $V_1 = I \times 6$ and $V_2 = I \times 2$

Therefore, $-9 + I \times 6 + I \times 2 + 3 = 0$

$I = \frac{6}{8}$

Consequently, $V_1 = I \times 6 = \frac{6}{8} \times 6 = \frac{9}{2}$ V.

Now, we will apply KVL to another "loop" shown below.



In which case, we have

$+9 - V_1 - V_o = 0$

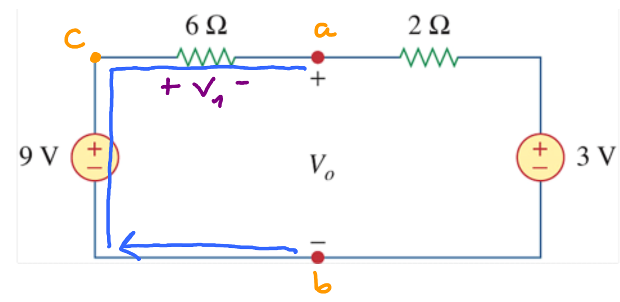
$V_o = 9 - V_1$

$= 9 - 9/2$

$= \frac{9}{2} = 4.5$ V

Alternatively, one can also consider the node voltages V_a and V_b. (The concept of node voltage is discussed in chapter 3. It is an important tool for a technique called "nodal analysis".)

Note that $V_o = V_b - V_a$.



Starting from node b, moving clockwise, we gain 9 V and then lose V_1 before we reach node a. Therefore,

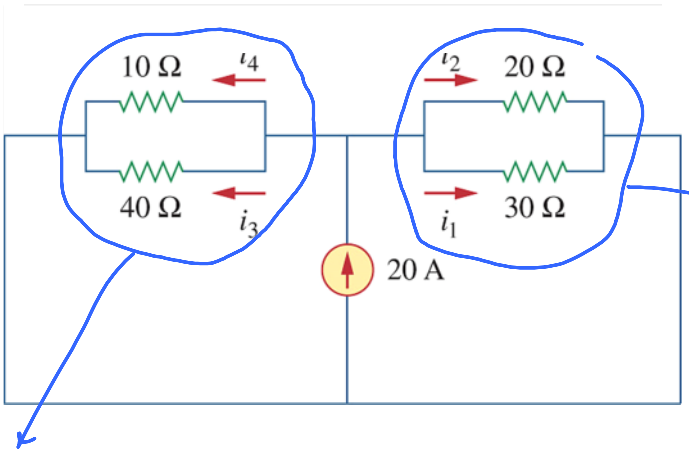
$V_b + 9 - V_1 = V_a$

$$V_b + 9 - V_1 = V_a .$$

So, $$V_a - V_b = 9 - V_1 .$$

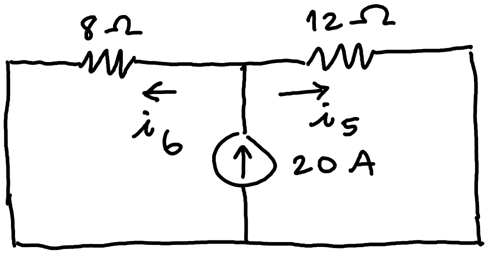
(CDF)

We will use the current divider formula.



→ parallel : $\frac{20\times30}{20+30} = \frac{60\cancel{0}}{5\cancel{0}} = 12\ \Omega$

parallel : $\frac{10\times40}{10+40} = \frac{40\cancel{0}}{5\cancel{0}} = 8\ \Omega$



So, 20 A is split into the 8 Ω and 12 Ω resistors.

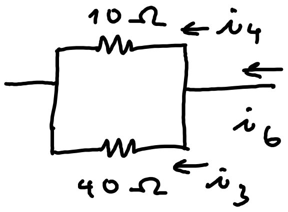
By CDF,

$$i_5 = \frac{\frac{1}{12}}{\frac{1}{12}+\frac{1}{8}} \times 20 = \frac{1}{\cancel{12}} \times \frac{\cancel{12}\times 8}{\cancel{12+8}} \times \cancel{20}$$

$$= 8A.$$

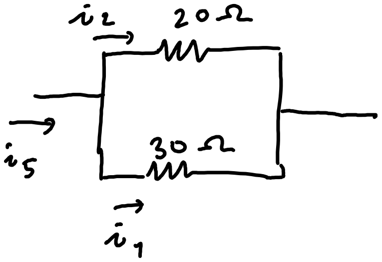
$i_6 = 20A - 8A = 12A.$ ← We can apply CDF to get i_6. However, it is easier to use KCL : $i_6 + i_5 - 20 = 0.$

The current i_6 is split again into i_3 and i_4.



By CDF,

The current i_5 is split again into i_1 and i_2.



By CDF,

By CDF,

$$i_3 = \frac{\frac{1}{4\cancel{0}}}{\frac{1}{1\cancel{0}}+\frac{1}{4\cancel{0}}} \times 12 = \frac{1}{\cancel{4}} \times \frac{\cancel{4}\times 1}{4+1} \times 12$$

$$= \frac{12}{5} = \frac{24}{10} = 2.4 \text{ A}.$$

$$i_4 \underset{\substack{\nearrow \\ \text{KCL}}}{=} i_6 - i_3 = 12 - 2.4 = 9.6 \text{ A}$$

By CDF,

$$i_1 = \frac{\frac{1}{30}}{\frac{1}{30}+\frac{1}{20}} \times 8 = \frac{1}{\cancel{30}} \times \frac{2\cancel{0}\times\cancel{30}}{2\cancel{0}+3\cancel{0}} \times 8$$

$$= \frac{16}{5} = \frac{32}{10} = 3.2 \text{ A}$$

$$i_2 \underset{\substack{\nearrow \\ \text{KCL}}}{=} i_5 - i_1 = 8 - 3.2 = 4.8 \text{ A}$$

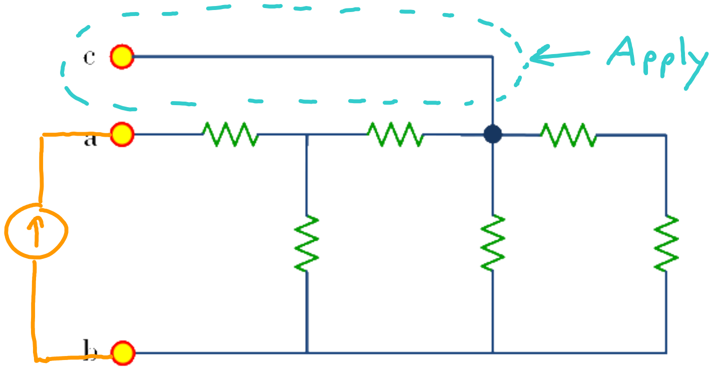
So,

$$\boxed{\begin{aligned} i_1 &= 3.2 \text{ A} \\ i_2 &= 4.8 \text{ A} \\ i_3 &= 2.4 \text{ A} \\ i_4 &= 9.6 \text{ A} \end{aligned}}$$

Resistor combination

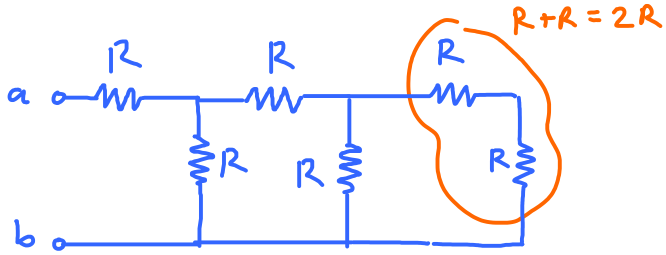
Thursday, January 29, 2015 12:28 PM

(a) To simplify the analysis, we first try to eliminate "hanging" branch(es) To do this, we connect a current source across the two terminals under consideration. Then, we use KCL to check whether there should be some (net) current flowing into a specipic brance of the circuit.



Applying KCL to this region, we know that there cannot be any (net) current going into this branch. So, this is a "hanging" branch which we can ignore.

From the point of view of terminals a-b, the circuit is



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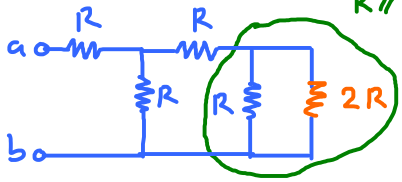
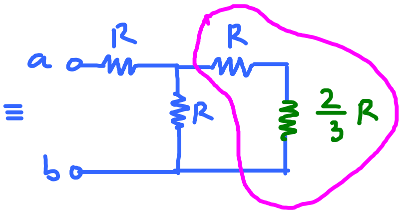


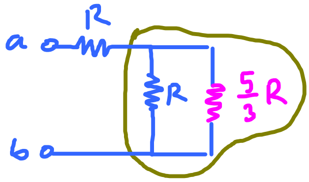
$$R /\!/ 2R = \frac{R \times 2R}{R + 2R} = \frac{2}{3}R$$

≡



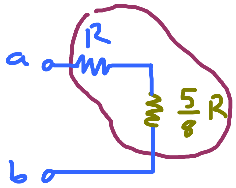
$$R + \frac{2}{3}R = \frac{5}{3}R$$

≡



$$R /\!/ (\tfrac{5}{3}R) = \frac{R \times \frac{5}{3}R}{R + \frac{5}{3}R} = \frac{5}{8}R$$

≡

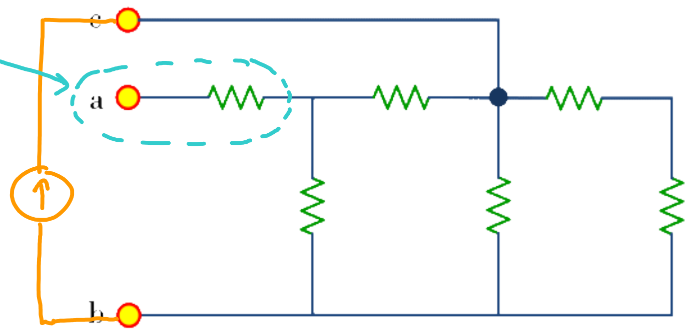


$$R + \frac{5}{8}R = \frac{13}{8}R$$

Therefore, $R_{ab} = \frac{13}{8}R$. Plugging in $R = 5\,\Omega$, we have $R_{ab} = \frac{65}{8} = 8.125\,\Omega$

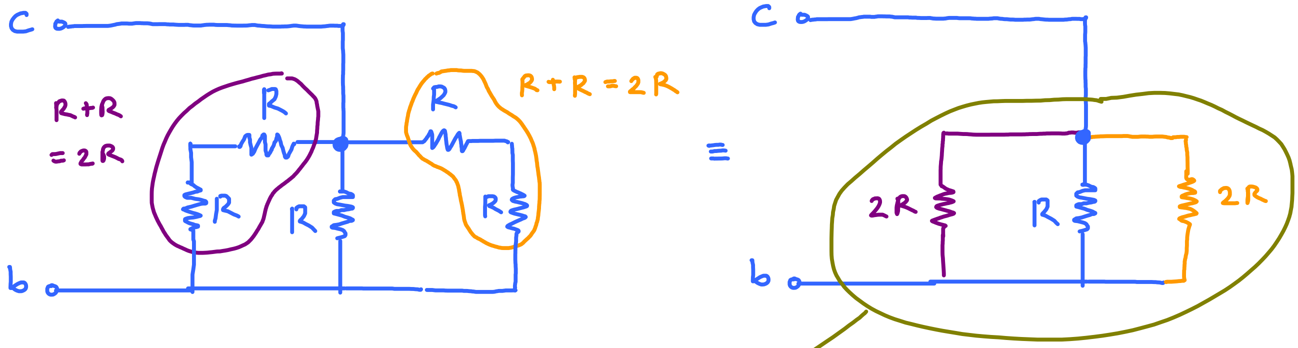
(b) To simplify the analysis, we first try to eliminate "hanging" branch(es) To do this, we connect a current source across the two terminals under consideration. Then, we use KCL to check whether there should be some (net) current flowing into a specipic brance of the circuit.

(net) current flowing into a specipic brance of the circuit.



Applying KCL to this region, we know that there cannot be any (net) current going into this branch. So, this is a "hanging" branch which we can ignore.

From the point of view of terminals b-c, the circuit is



$$R_{bc} = \frac{1}{\frac{1}{2R} + \frac{1}{R} + \frac{1}{2R}} = \frac{1}{\frac{4}{2R}} = \frac{R}{2}$$

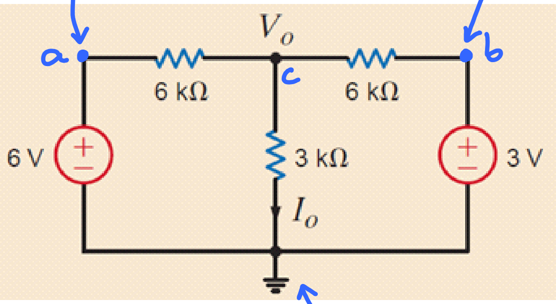
$$= \frac{5}{2} = 2.5\,\Omega$$

[Irwin Nelms, 2011, E3.7]

Thursday, January 29, 2015 1:17 PM

(1s) $V_a = 6V$

(1s) $V_b = 3V$



⓪ The reference node has already been specified here.

② At node c,

$$\frac{V_c - \overset{6V}{V_a}}{6k} + \frac{V_c - 0}{3k} + \frac{V_c - \overset{3V}{V_b}}{6k} = 0$$

③ $$\left(\frac{1}{6} + \frac{1}{3} + \frac{1}{6}\right) V_c - \frac{6}{6} - \frac{3}{6} = 0$$

$$(1 + 2 + 1)\, V_c - 6 - 3 = 0$$

$$V_c = \frac{9}{4}$$

④ $V_o = V_c = \frac{9}{4}$ V

$$I_o = \frac{V_c - 0}{3k} = \frac{9/4}{3k} = \frac{3}{4} \text{ mA}$$